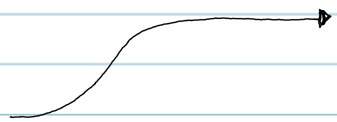
Section 2.4

"Logistics Functions"



$f(x) = \dfrac{N}{1 + A \cdot b^{-x}}$

N = limiting amt/value
A = involves initial amt
b = rate
$b < \varnothing\ \underline{1}$ "decay" (graph going down)
$b > \varnothing\ \underline{1}$ "growth" (graph going up)

y-int = $\dfrac{N}{1+A}$, when $x = 0$

$g(x) = \dfrac{100}{1 + 5(0.5)^{-x}}$

N = 100
A = 5
b = 0.5

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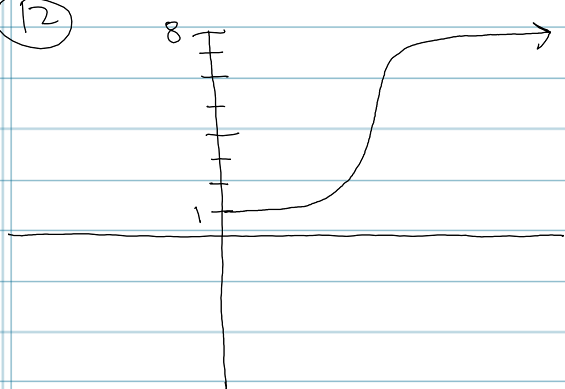

Match

A) $\dfrac{8}{1 + 7(2)^{-x}}$ $\dfrac{8}{1+7} = 1$ ✓

B) $\dfrac{8}{1 + 3(2)^{-x}}$ $\dfrac{8}{1+3} = 2$

C) $\dfrac{6}{1 + 11(5)^{-x}}$ ← limiting value wrong

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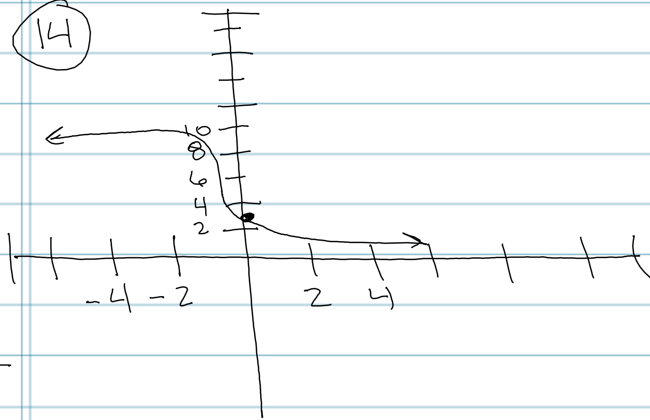

*Test

A) $\frac{10}{1+3(1.01)^{-x}}$ $\quad \frac{10}{4} = 2.5$

$\cancel{\text{B)}}$ $\frac{8}{1+7(0.1)^{-x}}$ — limiting value wrong

C) $\frac{10}{1+3(0.1)^{-x}}$ $\quad \frac{10}{4} = 2.5$

$b<1$ ✓

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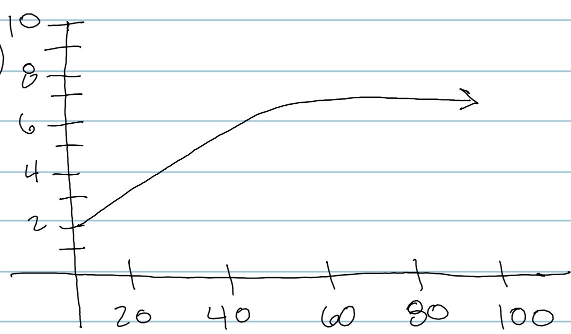

A) $\frac{14}{2+5(15)^{-x}}$ $\quad \frac{7}{1+\frac{5}{2}(15)^{-x}}$

$\cancel{\text{B)}}$ $\frac{14}{1+13(1.05)^{-x}}$

C) $\frac{14}{2+5(1.05)^{-x}}$ $\quad \frac{7}{1+\frac{5}{2}(1.05)^{-x}}$ ✓

30 Coca-Cola Sales in Russia

$f(x) = \frac{N}{1 + A \cdot b^{-x}}$

1993 $\rightarrow t = 0$ $\quad f(0) = 2$

$f(2) = 4$

2 years doubling time $\quad t = 2$

$N = 100$

goal $\rightarrow$ year (t) $\rightarrow$ 50 servings

Setup

$f(x) = \frac{100}{1 + A \cdot b^{-x}}$ $\quad * \frac{N}{1+A} = $ y-int

$1 + A = \frac{100}{2}$ $\quad 2 = \frac{100}{1+A}$

$1 + A = 50$

$A = 49$

Next, $(2, 4)$

$f(x) = \frac{100}{1 + 49 \cdot b^{-2}} = 4$

$4(1 + 49 \cdot b^{-2}) = 100$

$1 + 49b^{-2} = 25$

$-1 \qquad -1$

$49b^{-2} = 24$

$b^{-2} = \frac{24}{49}$

$b^2 = \frac{49}{24}$ "Take sq. root"

$b = \sqrt{\frac{49}{24}} = 1.4289$

Cont'd : "Russian Cola"

$$f(x) = \frac{100}{1+49(1.4289)^{-x}}$$

They want to know 50 servings

$$50 = \frac{100}{1+49(1.4289)^{-x}}$$

$$\frac{50(1+49\cdot(1.4289)^{-x}}{50} = \frac{100}{50}$$

$$1+49(1.4289)^{-x} = 2$$

$$-1 \qquad -1$$

$$49(1.4289)^{x} = 1$$

$$1.4289^{-x} = \frac{1}{49}$$

$$\ln(1.4289)^{-x} = \ln\left(\frac{1}{49}\right)$$

$$-x\cdot\ln(1.4289) = \ln\left(\frac{1}{49}\right)$$

$$x = \frac{\ln(1/49)}{-\ln(1.4289)}$$

$x = 10.9$ years

1993 = 0
1994 = 1
" "
2003 = 10 Towards the end of 2003

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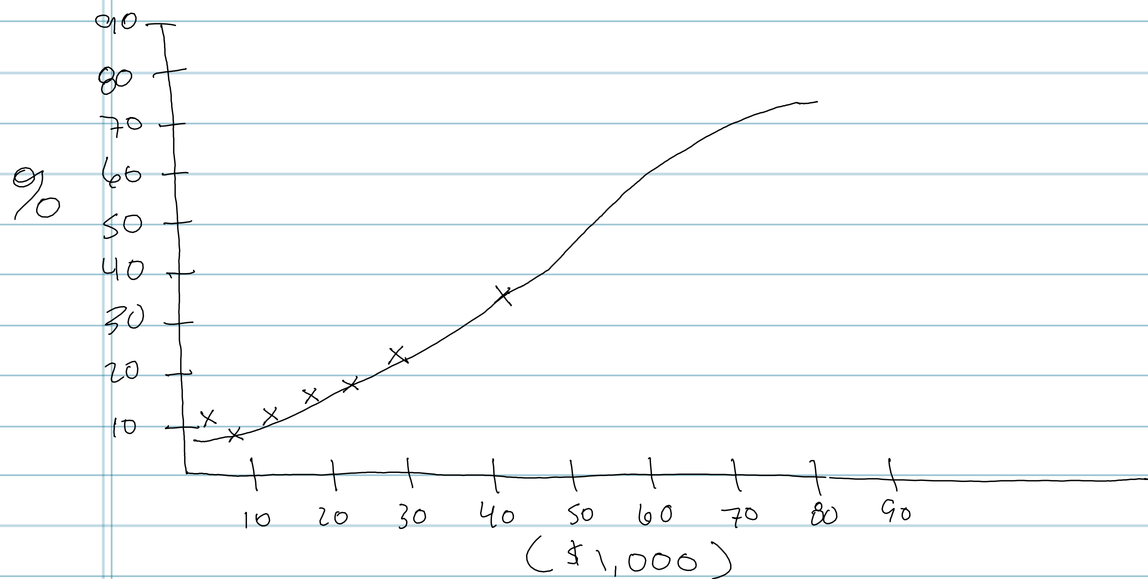
26 Internet use

Logistic model:

$$P(x) = \frac{64.2}{1 + 9.6(1.06)^{-x}} \text{ percent}$$



a) 62%

b) $\left(\frac{N}{1+A}\right)b^x$ for small values $\left(\frac{64.2}{1+9.6}\right)(1.06)^x$

c) $60,000 @ 50%